

QUANT-TOOL KIT: SOME IMPORTANT THINGS TO REVISE BEFORE CAT

**NUMBERS**

**Types of Numbers:**

Natural numbers (Positive integers)	:	1, 2, 3, 4,....
Whole numbers (Non-negative integers)	:	0, 1, 2, 3,....
Negative integers	:	- 1, - 2, - 3,....
Integers	:	...., -2, - 1, 0,1, 2,.....
Even numbers	:	...., - 2, 0, 2, 4,.... (2n)
Odd numbers	:	....., -3, -1,1,3,... (2n + 1)
Prime numbers (exactly 2 factors)	:	2, 3, 5, 7, 11,....
Composite numbers (more than 2 factors)	:	4, 6, 8, 9,10,....
Perfect numbers (Sum of all the factors is twice the number)	:	6, 28, 496, ..
Co-primes (relative primes) (Two numbers whose HCF is 1)	:	2 & 3, 8 & 9,..
Twin primes (Two prime numbers whose difference is 2) :		3 & 5, 5 & 7,.
Rational numbers ( $\frac{p}{q}$ form, p & q are integers, q ≠ 0)	:	$\frac{2}{3}$ , $\frac{3}{2}$ , 2, 0.5,..
Irrational numbers (which cannot be represented in the form of a fraction)		
: $\sqrt{2}$ , $\sqrt[3]{5}$ , e, $\pi$ , 0.231764735...		

**Pure recurring decimal to fraction conversion**

Ex.  $0.ababab \dots = \frac{ab}{99}$

**Mixed recurring decimal to fraction conversion**

Ex.  $0.abcbcbc \dots = \frac{abc - a}{990}$

- 1 is the neither prime, nor composite.
- 2 is the only even prime.
- If x & y are two integers, then (x + y) ! is divisible by x !. y!
- The product of 'n' consecutive numbers is divisible by n!.
- **Divisibility of powers:**

	n is even	n is odd.
$x^n - y^n$	divisible by (x-y)(x+y)	divisible by (x-y)
$x^n + y^n$	----	divisible by (x+y)

**Some Important points:**

- Every number 'N' can be written as  $N = a^p \times b^q \times c^r \dots$  (a, b, c,.... are prime numbers.)
- If p, q, r ..... are even, 'N' is a perfect square.
- If p, q, r are multiples of 3, 'N' is a perfect cube.
- Number of factors of  $N = (p+ 1) (q + 1) (r + 1) \dots$
- Sum of the factors of  $N = \left( \frac{a^{p+1} - 1}{a - 1} \right) \left( \frac{b^{q+1} - 1}{b - 1} \right) \dots$
- Number of co – primes of 'N' , which are less than N =  $N(1-1/a)(1-1/b) \dots$

- Sum of these co-primes =  $\frac{N}{2} \times N (1 - 1/a) (1 - 1/b) \dots$
- Numbers of ways of writing 'N' as a product of 2 co-primes =  $2^{n-1}$ , n is the number of different prime numbers in 'N'
- If n is a prime number,  $(n-1)! + 1$  is divisible by n.
- If n is a natural number and p is a prime number, then  $(n^p - n)$  is divisible by p
- The last digit of the powers of 2,3,7,8 repeats after every 4<sup>th</sup> power.
- The last digit of any power of 0,1, 5,6 is always 0,1,5, 6 respectively.
- The last digit of the powers of 4 and 9 repeats after every 2<sup>nd</sup> power.

**Divisibility Rules:**

2 or 5 → check last digit. If it's divisible by 2 or 5

4 or 25 → check the last two digits. If it's divisible by 4 or 25

3 or 9 → check the sum of the digits. If it's divisible by 3 or 9

11 → check the difference of (sum of the digits in the even places) and (the sum of digits in odd places).... If it's divisible by 11

**LCM / HCF**

$a \times b = \text{LCM} (a, b) \times \text{HCF} (a, b)$

$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

IMPORTANT RESULTS		
S.No.	Type of Problem	Approach of Problem
1.	Find the GREATEST NUMBER that will <i>exactly</i> divide x, y, z.	Required number = H.C.F. of x, y, and z (greatest divisor).
2.	Find the GREATEST NUMBER that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = H.C.F. of $(x - a)$ , $(y - b)$ and $(z - c)$ .
3.	Find the LEAST NUMBER which is <i>exactly</i> divisible by x, y and z.	Required number = L.C.M. of x, y and z (least multiple).
4.	Find the LEAST NUMBER which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that $(x - a) = (y - b) = (z - c) = K$ (say). ∴ Required number = (L.C.M. of x, y and z) - (K).
5.	Find the LEAST NUMBER which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (L.C.M. of x, y and z) + r.
6.	Find the GREATEST NUMBER that will divide x, y and z leaving the same remainder in each case.	Required number = H.C.F. of $(x - r)$ , $(y - r)$ and $(z - r)$ .

**AVERAGE**

- Average =  $\frac{\text{Sum of items}}{\text{Number of items}}$
- Weighted average =  $x_1, x_2, \dots, x_n$ , which are in the ratio  $r_1 : r_2 : \dots : r_n$  is.  

$$\frac{r_1x_1 + r_2x_2 + \dots + r_nx_n}{r_1 + r_2 + \dots + r_n}$$
- The average of consecutive numbers or the numbers which are in A.P is the middle number or the average of the first and the last number.
- If each number is increased / decreased/ multiplied/ divided by a number 'k', the average is also increased/ decreased/ multiplied/ divided by k.

### PERCENTAGE

% change =  $\frac{\text{F.V.} - \text{I.V.}}{\text{I.V.}} \times 100$       Where F.V. =Final value    I.V.= Initial value

#### Percentage to Fraction conversions:

$\frac{1}{1} = 100\%$	$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33.33\%$	$\frac{1}{4} = 25\%$
$\frac{1}{5} = 20\%$	$\frac{1}{6} = 16.67\%$	$\frac{1}{7} = 14.28\%$	$\frac{1}{8} = 12.5\%$
$\frac{1}{9} = 11.11\%$	$\frac{1}{10} = 10\%$	$\frac{1}{11} = 9.09\%$	$\frac{1}{12} = 8.33\%$
$\frac{1}{13} = 7.69\%$	$\frac{1}{14} = 7.14\%$	$\frac{1}{15} = 6.66\%$	$\frac{1}{16} = 6.25\%$

- Profit = S.P – C.P
- % profit =  $\frac{\text{Profit}}{\text{C.P}} \times 100$
- Discount = M.P – S.P      C.P = Cost price,
- % discount =  $\frac{\text{Discount}}{\text{M.P}} \times 100$       M.P. = Marked price,      S.P = Selling price

### RATIO & PROPORTION

If  $a/b = c/d = e/f$ , then

$$a/b = c/d = e/f = \frac{a+c+e}{b+d+f} = \frac{a+c+e}{b+d+f} = \frac{k_1a+k_2c+k_3e}{k_1b+k_2d+k_3f}$$

#### Partnership & Share:

If there is profit in the business run by two partners A and B then,

$$\frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Amount of A's investment} \times \text{No. of months of A's investment}}{\text{Amount of B's investment} \times \text{No. of months of B's investment}}$$

**Proportion:**

1. If x is directly proportional to y,

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

2. If x is inversely proportional to y,  $x_1y_1 = x_2y_2$

**Alligation:**

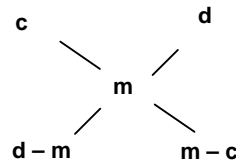
$Q_c$  → Cheaper quantity

$Q_d$  → Dearer quantity

c → C.P. of unit qty of 1<sup>st</sup> constituent.

d → C.P. of unit qty of 2<sup>nd</sup> constituent.

m → Mean Cost price of unit qty of mixture



$$\frac{Q_c}{Q_d} = \frac{d - m}{m - c}$$

Gives us the ratio of quantities in which the two ingredients should be mixed to get the mixture.

**Mixtures:**

If a vessel contains “x” litres of milk and if “y” litres be withdrawn and replaced by water, then if “y” litres of the mixture be withdrawn and replaced by water, and the operation repeated ‘n’ times in all, then :

$$\frac{\text{Milk left in vessel after nth operation}}{\text{Initial quantity of Milk in vessel}} = \left[ \frac{x - y}{x} \right]^n = \left[ 1 - \frac{y}{x} \right]^n$$

**ALGEBRA**

**Laws of indices:**

→  $(a^m)^n = a^{mn}$

→  $a^{-n} = \frac{1}{a^n}$

→  $(ab)^m = a^m b^m$

→  $a^{\frac{1}{n}} = \sqrt[n]{a}$

**Logarithm:**

1.  $\log_a 0 = \text{not defined}$

2.  $\log_a 1 = 0$

3.  $\log_a m + \log_a n = \log_a mn$

4.  $\log_a m^n = n \log_a m$

5.  $\log_a m - \log_a n = \log_a \left( \frac{m}{n} \right)$

6.  $\log_{ba} = \frac{\log_c a}{\log_c b}$

7.  $\log_a a = 1$

8.  $\log_{ba} = \frac{1}{\log_a b}$

9.  $a^{\log_a m} = m$

10.  $\log_{a^n} (m) = \frac{1}{n} \cdot \log_a m$

**Remainder Theorem:**

If  $f(x)$  is divided by  $x - a$ , the remainder is  $f(a)$

**Quadratic equations:**

The general form of Quadratic Equation is  $ax^2 + bx + c = 0$

The roots are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of the roots =  $-\frac{b}{a}$

Product of the roots =  $\frac{c}{a}$

**Nature of the roots:**

If  $b^2 - 4ac < 0$ , then roots are Imaginary

If  $b^2 - 4ac = 0$ , the roots are real and equal

If  $b^2 - 4ac > 0$ , the roots are real and distinct

If  $b^2 - 4ac$  is perfect square, the roots are rational

**Inequalities/ Max. Min.**

If  $a > b$ ,  $a \pm c > b \pm c$

If  $a > b \Rightarrow ac > bc$  if  $c > 0$  and  $ac < bc$  if  $c < 0$

If  $a > b \Rightarrow 1/a < 1/b$

$\rightarrow a^2 + b^2 + c^2 \geq ab + bc + ca$

$\rightarrow 2 \leq \left(1 + \frac{1}{x}\right)^x \leq 3.$

$\rightarrow a/b + b/c + c/a \geq 3$

$\rightarrow a^4 + b^4 + c^4 + d^4 \geq 4abcd$

$\rightarrow a^3 + b^3 + c^3 \geq 3abc$

$\rightarrow |a + b| \leq |a| + |b|$

$\rightarrow |a - b| \geq |a| - |b|$

$\rightarrow A.M \geq G.M \geq H.M.$       A.M = Arithmetic mean,    G.M = Geometric mean,    H.M = Harmonic mean

$\rightarrow$  For any Quadratic expression;  $ax^2 + bx + c$ ,

The min. or max. value will come at  $x = -b/2a$

If  $a < 0$ , the value is maximum.

$a > 0$ , the value is minimum.

The min/ max value =  $\frac{4ac - b^2}{4a}$ .

**Progressions:**

A.M =  $\frac{a_1 + a_2 + \dots + a_n}{n}$  ,      A.M is Arithmetic mean

G.M =  $\sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$       G.M is the Geometric mean

H.M =  $\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}$       H.M is the Harmonic mean

**A.P.:**

a, a + d, a + 2d, ..... ( a = first term, d = common difference)

nth term,  $T_n = a + (n - 1) d$

Sum of first n terms:  $S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} (\text{First term} + \text{Last term})$

**G.P.:**

a. ar, ar<sup>2</sup>, ..... ( a= first term, r =- common ratio)

nth term,  $T_n = a.r^{n-1}$ ,

Sum of first n terms:  $S_n = \frac{a(r^n - 1)}{r - 1}$

Sum of infinite terms of G.P =  $\frac{a}{1-r}$

→  $\sum n = \frac{n(n+1)}{2}$  (  $\sum n$  is the sum of the first n natural numbers)

→  $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$  (  $\sum n^2$  is the sum of the squares of the first n natural numbers)

→  $\sum n^3 = \left[ \frac{n(n+1)}{2} \right]^2$  (  $\sum n^3$  is the sum of the cubes of the first n natural numbers)

**TIME & WORK**

$$\frac{M_1 D_1 H_1 E_1}{W_1} = \frac{M_2 D_2 H_2 E_2}{W_2}$$

M = Number of men,      D = Number of days

H = Number of hours,      E = Efficiency                      W = Amount of work

**TIME & DISTANCE**

Distance = Speed × Time

**Relative speed of A & B:**

Same direction :       $V_A - V_B$

Opposite direction :       $V_A + V_B$  (  $V_A$  is the speed of A,  $V_B$  is the speed of B)

**Resultant Speed:**

Same direction :      B + W

Opposite direction:      B – W (B is the speed of boat, W is the speed of water)

**Average speed =  $\frac{\text{Total distance}}{\text{total time}}$**

**SETS**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

**INTEREST**

$S.I = \frac{Ptr}{100}$  (S.I = Simple Interest, P = principle, t = time, r = rate of interest)

$C.I = P \left( 1 + \frac{r}{100} \right)^n - P$  (C.I = Compound Interest)

Amount = principle + interest

**PERMUTATIONS / COMBINATIONS**

- ${}^n C_r = \frac{n!}{(n-r)! r!}$        ${}^n C_0 = {}^n C_n = 1, {}^n C_1 = n$
- ${}^n P_r = \frac{n!}{(n-r)!}$
- ${}^n P_r = {}^n C_r \times r!$        ${}^n P_0 = 1, {}^n P_1 = n, {}^n P_n = n!$
- r things can be selected from n things in  ${}^n C_r$  ways.
- r things can be arranged in n places in  ${}^n P_r$  ways.
- n things can be placed in n places in n! ways.
- ${}^n C_0 + {}^n C_1 + \dots + {}^n C_n = 2^n$ .
- n persons can sit around the circular table in (n – 1)! ways

**PROBABILITY**

Probability =  $\frac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes}}$

- $P(\bar{A}) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- Odds in favor of A =  $P(A) : P(\bar{A})$
- Odds against A =  $P(\bar{A}) : P(A)$
- If A and B are independent events  $P(A \cap B) = P(A) \times P(B)$

**CLOCKS**

The angle between the hands =  $\left| 30H - \frac{11}{2}M \right|^\circ$

(Where H → Hour reading & M → Minute reading)

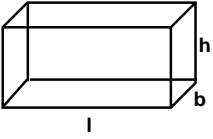
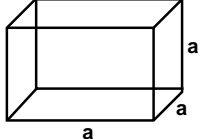
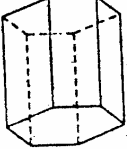
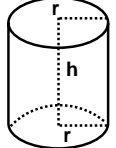
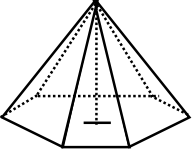
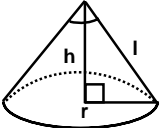
- The hands will coincide once in every  $65\frac{5}{11}$  minutes.
- In 12 hours, the hands will coincide 11 times.
- The hands will make an angle 'θ' ( $0 < \theta < 180$ ), 22 times in 12 hours.

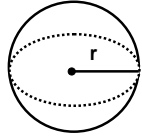

**AREA**

Square	$a^2$	( a is side)
Rectangle	$l b$	( l = length, b = breadth)
Parallelogram	$ab \sin \theta = ah$	( a and b are two sides and $\theta$ is the angle between them, h is the height)
Rhombus	$\frac{1}{2} d_1 d_2$	( $d_1$ , $d_2$ are the diagonals)
Trapezium	$\frac{1}{2} (a + b) h$	( a and b are the two parallel sides and h is the distance between them)
Triangle	$\frac{1}{2} bh$	( b is the base, h is the height)
	$= \frac{1}{2} ab \sin \theta$	( a and b are two sides and $\theta$ is the angle between them )
	$= \sqrt{s(s-a)(s-b)(s-c)}$ , $s = \frac{a+b+c}{2}$	( a , b , c are the three sides )
Equilateral triangle	$\frac{\sqrt{3}}{4} a^2 = \frac{h^2}{\sqrt{3}} \left( h = \frac{\sqrt{3}}{2} a \right)$	( a is the side, h is the height )
Circle	$\pi r^2$	( r is the radius)
Sector	$\frac{\theta}{360} \times \pi r^2$	( $\theta$ is the angle made by the sector at the center ) radiusRegular
Regular hexagon	$6 \times \frac{\sqrt{3}}{4} a^2$	( a is side)
Cyclic Quadrilateral	$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$ , $s = \frac{a+b+c+d}{2}$	( a , b , c , d are sides)



**VOLUME**

S. No	Nature of the solid	Shape of the solid	Lateral/ curved surface area	Total surface area	Volume	Abbreviations Used
1.	Cuboid		$2h(l + b)$	$2(lb + bh + lh)$	$lbh$	l = length b = breadth h = height
2.	Cube		$4a^2$	$6a^2$	$a^3$	a = length of edge
3.	Right prism		(perimeter of base) × Height	2 (area of one end) + lateral surface area	Area of base × height	
4.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height of the cylinder
5.	Right pyramid		$\frac{1}{2}$ (Perimeter of the base) × (slant height)	Area of the base + lateral surface area	$\frac{1}{3}$ (Area of base) × height	
6.	Right circular cone		$\pi r l$	$\pi r(l + r)$	$\frac{1}{3} \pi r^2 h$	h = height r = radius l = slant height

7.	Sphere		—	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = radius
8.	Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\pi r^3\right)$	r = radius

**GEOMETRY**

**Diagnol of Cube and Cuboid:**

The length of diagonal of a cuboid =  $\sqrt{l^2 + b^2 + h^2}$

The length of a diagonal of a cube =  $a\sqrt{3}$

**Polygons:**

Sum of all external angles of any polygon =  $360^\circ$

Sum of all internal angles =  $(n - 2) 180^\circ$

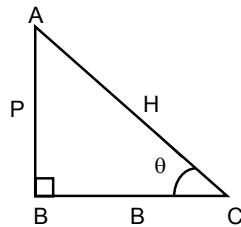
Number of diagonals in a polygon =  ${}^nC_2 - n = \frac{n(n-3)}{2}$

**Pythagoras Theorem**

$$AB^2 + BC^2 = AC^2$$

**Basic Trigonometric Ratios**

In a right triangle ABC, if  $\theta$  be the angle between AC & BC.



If  $\theta$  is one of the angle other then right angle, then the side opposite to the angle is perpendicular (P) and the sides containing the angle are taken as Base ( B) and the hypotenuse (H). In this type of triangles, we can have six types of ratios. These ratios are called trigonometric ratios.

$$\sin \theta = \frac{P}{H}, \quad \cos \theta = \frac{B}{H}, \quad \tan \theta = \frac{P}{B}$$

$$\operatorname{Cosec} \theta = \frac{H}{P}, \quad \operatorname{Sec} \theta = \frac{H}{B}, \quad \operatorname{Cot} \theta = \frac{B}{P}$$

**Cosine Rule**

In triangle ABC with sides a, b, c, we have the following rules;

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

( a, b, c are the sides opposite to the angles A, B, C respectively )

**Similar Triangles**

If  $\triangle ABC$  and  $\triangle DEF$  are similar.

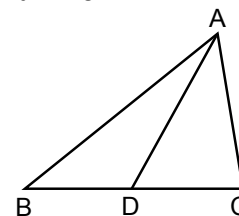
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{\text{Area of } ABC}{\text{Area of } DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

**Centroid:**

- (a) The point of intersection of the medians of a triangle. (Median is the line joining the vertex to the mid-point of the opposite side).
- (b) The centroid divides each median from the vertex in the ratio 2 : 1.
- (c) To find the length of the median we use the theorem of Apollonius.

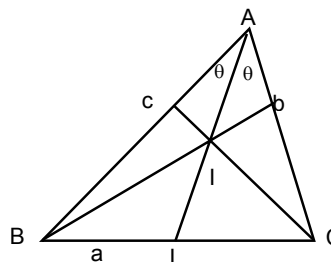
$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$



**Incentre:**

This is the point of intersection of the internal bisectors of the angles of a triangle.

- (a)  $\frac{BL}{LC} = \frac{AB}{AC}$
- (b)  $\frac{AI}{IL} = \frac{b+c}{a}$

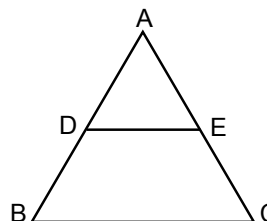


**Mid-point Theorem**

A line joining the mid points of any two sides of a triangle must be parallel to the third side and equal to half of that (third side).

In the adjacent triangle ABC, if D and E are the respective mid-points of sides AB & AC, then

$$DE \parallel BC \text{ and } DE = \frac{1}{2} BC$$

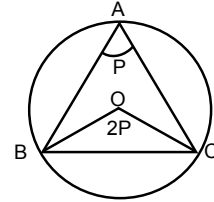


**Properties of a Circle**

1. If two chords of a circle are equal, their corresponding arcs have equal measure.
2. Degree measure of an arc is the angle subtended at the centre. Equal arcs subtend equal angles at the center.
3. A line from centre and perpendicular to a chord bisects the chord.
4. Equal chords of a circle are equidistant from the centre.
5. When two circles touch, their centres and their point of contact are collinear.
6. If the two circles touch externally, the distance between their centres is equal to sum of their radii.
7. If the two circles touch internally, the distance between the centres is equal to difference of their radii.
8. Angle at the centre made by an arc is equal to twice the angle made by the arc at any point on the remaining part of the circumference.

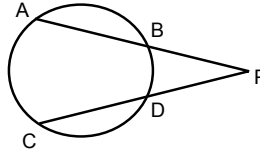
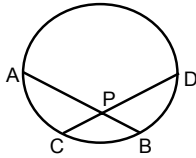
Let O be the centre of the circle.

$$\angle BOC = 2 \angle P,$$

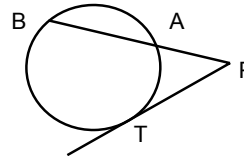


9. There can be one and only one circle through three non-collinear points.
10. The angle inscribed in a semicircle is  $90^\circ$ .
11. If two chords AB and CD intersect externally or internally at P, then

$$PA \times PB = PC \times PD$$

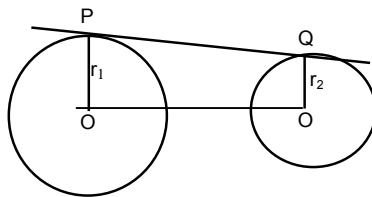


12. If PAB is a secant and PT is a tangent, then  $PT^2 = PA \times PB$



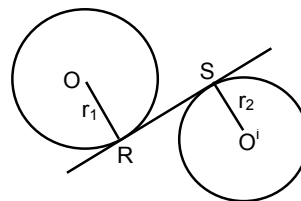
13. The length of the direct common tangent (PQ)

$$= \sqrt{(\text{The distance between their centres})^2 - (r_1 - r_2)^2}$$



14. The length of the transverse common tangent (RS)

$$= \sqrt{(\text{The distance between their centres})^2 - (r_1 + r_2)^2}$$



**CO-ORDINATE GEOMETRY**

**Distance formula:**

If A ( $x_1, y_1$ ) and B ( $x_2, y_2$ ) be two points, then

$$| AB | = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In particular, distance of a point P ( $x, y$ ) from O ( $0, 0$ ) is  $| OP | = \sqrt{x^2 + y^2}$

**Section formula:**

The point which divides the join of two distinct points A ( $x_1, y_1$ ) and B ( $x_2, y_2$ ) in the ratio  $m_1 : m_2$  internally, has

the co-ordinates  $\left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$ ,  $m_1 \neq 0, m_2 \neq 0, m_1 + m_2 \neq 0$

and externally, is

$$\left( \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right) \quad m_1 \neq 0, m_2 \neq 0, m_1 - m_2 \neq 0$$

In particular, the mid-point of the segment joining A ( $x_1, y_1$ ) and B ( $x_2, y_2$ ) has the co-ordinates

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Centroid and Incentre formulae:**

**Centroid:** It is the point of intersection of the medians of a triangle.

**Incentre:** It is the point of intersection of the internal angle bisectors of the angles of a triangle.

If A ( $x_1, y_1$ ), B ( $x_2, y_2$ ) and C ( $x_3, y_3$ ) be the vertices of a triangle, then its **centroid** is given

by  $\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$ , and the **incentre** by  $\left( \frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$ .

Where  $a = | BC |$ ,  $b = | CA |$  and  $c = | AB |$ .

**Equation of a line:**

**One point form**

Equation of a line (non-vertical) through the point ( $x_1, y_1$ ) and having slope  $m$  is

$$y - y_1 = m (x - x_1).$$

**Two-point form**

Equation of a line (non-vertical) through the points ( $x_1, y_1$ ) and ( $x_2, y_2$ ) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

**Intercept form**

Equation of a line (non-vertical) with slope  $m$  and cutting off intercepts  $a$  and  $b$  from the x-axis and y-axis

respectively is  $\frac{x}{a} + \frac{y}{b} = 1$ .

**Some important points:**

- (a) Slope of a line parallel of x-axis is zero.
- (b) Slope of a line parallel to y-axis is not defined.
- (c) Slope of a line equally inclined to equal the axis is  $-1$  or  $1$ .
- (d) Slope of a line making equal intercepts on the axis is  $-1$ .
- (e) Slope of the line through the points A  $(x_1, y_1)$  and B  $(x_2, y_2)$  is  $\frac{y_2 - y_1}{x_2 - x_1}$
- (f) Slope of the line  $ax + by + c = 0$ ,  $b \neq 0$ , is  $-\frac{a}{b}$ .
- (g) Slopes of two parallel (non-vertical) lines are equal.
- (h) If  $m_1$  and  $m_2$  be the slopes of two perpendicular lines (which are oblique), then  $m_1 m_2 = -1$ .
- (i) Length of perpendicular from the point  $(x_1, y_1)$  to the line  $ax + by + c = 0$  is

$$L = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

- (j) Distance between parallel lines;  $ax + by + c = 0$  and  $ax + by + d = 0$

$$\frac{|c - d|}{\sqrt{a^2 + b^2}}$$

**Area of triangle:**

If A  $(x_1, y_1)$ , B  $(x_2, y_2)$  and C  $(x_3, y_3)$  are the vertices of a triangle then its area is equal to  $= \frac{1}{2} \text{ mod of } \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$